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Raman, in a portrait painted in the 1950s.
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Perceptual, Acoustical, and Musical Aspects of the Tambūrā Drone

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The basso continuo principle, as embodied in Rameau's theory of functional harmony, was paralleled by the introduction of drone instruments in the classical music of India. In order to understand how these two systems are tied together in human music perception, we studied the role of tambūrā interactions with North Indian rāgs played on the sitār. Raman (1914–1922) had applied his theory of discontinuous wave motion to mechanical and musical properties of the strings of the violin. He noted the remarkable, powerful harmonic series that arose from the nonlinear interaction of the tambūrā string and grazing contact with its curved bridge. We analyzed the waveforms of the most common drone tunings. Each of the four strings was played with and without juari ("life-giving" threads). The upward transfer and spread of energy into higher partials imparts richness to tambūrā tones and underlies the use of different drone tunings for different rāgs. Specific notes of rāg scales are selectively and dynamically enhanced by different drone tunings. Based on coincident features of spectral and musical scale degrees, we computed an index of spectral complexity of the interactions of tambūrā tunings with rāg scales. We speculate that the use of juari contributes to stable pitch centers, implied scale modulation, and an improvisational flexibility.

Introduction

Like Raman and Helmholtz before him, we seek to relate the physics of music to the perception of the listener. To metaphysical-aesthetical critics who rejected his theory of music as too coarsely mechanical, Helmholtz replied, ". . .that I cannot think I have undervalued the artistic emotions of the human kind . . . by endeavouring to establish the physiological facts upon which esthetic feeling is based" (1877/1954, p. vii). Raman said of his work on the mechanical theory of the vibrations of bowed strings that, "emphasis is laid upon the cases which are of practical interest in music" (1918, p. 3).

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Before Raman

BEFORE HELMHOLTZ

A fine, succinct history of violin research is given by Hutchins (1983). Table 1 is a very brief synopsis that shows some of the key contributions to theory and experiments on musical strings between 1625 and 1918. It took some 100 years to go from knowing that tonal pitch was related to the length, tension, and unit mass of the string to having a dynamic solution of the motion of a vibrating string (by Taylor, ca. 1720). Another 100 years later in 1819, Savart, using a cogwheel frequency standard, showed how to relate nodal and antinodal regions to specific frequencies of resonance of violin plates. He found that the major role of the violin soundpost was to alter vibrational modes of top and back plates, not to transmit vibrations between them. The next, giant step was made in 1862 by Helmholtz, to whom we now turn.

HELMHOLTZ

It was Helmholtz's stated aim to connect the boundaries of physical and physiological acoustics on the one side to musical science and aesthetics on the other. He was a gifted experimenter and theorist. He devised acoustic resonators for analysis of partials of complex sounds. By means of electromagnetic tuning forks he synthesized vowel sounds and showed that difference of phase made no difference to timbre.

Helmholtz studied plucked and struck strings and provided mathematical solutions of their motion. He was not able to give a complete mechanical theory of the motion of strings excited by the violin bow because "the mode in which the bow affects the motion of the string is unknown" (1862, p. 80).

From his observations of the vibrational form of individual points in a violin string, Helmholtz was able to calculate the whole motion of the string and the intensity of the upper partial tones. To see the form of vibration, he devised the vibration microscope after an idea of Lissajou. In essence, a white grain of starch was fastened to a blackened violin string. An electric fork vibrated a doublet lens vertically against the horizontal motion of the bowed string. The resultant motion seen by Helmholtz was a Lissajou figure with cusps (Figure 1, B & C). (What Helmholtz saw you can see on an oscilloscope by driving its beam horizontally with a sine wave and vertically with a sawtooth of the same frequency.) It was easy to infer the vibrational forms and see that they were essentially different from a simple sinusoid (Figure 1D, A & B). The wave is a sawtooth. The two periods of time into which the waveform is divided are in the same ratio as the two sections of

TABLE 1

Some Early Key Workers on the Properties of Musical Strings, Their Methods, Experiments and Major Findings

Worker	Methods: Mathematics	Methods: Experiments	Major Findings
		Strings	
Galileo (1564–1642)	Arithmetic, 1638	Iron chisel scrapes on brass plate	Pitch increased with scraping speed
Mersenne (1588–1648)	Arithmetic, 1625	Observed stretched string	Pitch of tone nearly f = (k/Length) × sqrt(Tension/Mass)
Hooke (1633–1703)	Arithmetic,	Vibrated cardboard membrane by cogwheel	Related frequency to pitch
Taylor (1685–1731)	Calculus	Dynamical solution of vibrating string	Verified work of Galileo & Mersenne
Duhamel (1797–1872)	Calculus, 1841	Theorized on duality of vibration period	Sticking and slipping during bowing
		Violins & Musical Strings	
Savart (1791–1841)	Arithmetic, 1819	Measured frequency by cogwheel; saw Chladni patterns on bowed violin plates	Related nodal and antinodal regions to specific frequencies of resonance
Helmholtz (1821–1894)	Fourier transforms; Kinematic theory of strings, 1862	Tuned resonators & electric forks; saw Lissajou figures in vibration microscope of his own devising	Observed: harmonics of complex tones; sawtooth wave of the bowed string. Computed the motion of the whole string
Lord Rayleigh (1842–1919) Nobel Prize, 1904	Electrical circuit-theory analogies, 1877	Used wide-ranging theory in exploring vibrations of bells, membranes, plates, & shells	Unified foundations of vibrating systems and acoustics
Raman (1888–1970) Nobel Prize, 1930	Fourier series; dynamical theory of the bowed string, 1911–1918	Devised mechanical bowing machine & moving-slit camera for observing motion of the whole string interacting with bow	Showed that minimum bowing force varied with speed of bow but inversely as square of distance of bow from bridge; showed how "wolf note" arose from energetics of bowed fundamental and body resonance

NOTE. See Hutchins (1983).

the string that lie on either side of the observed point. The string velocity (the time derivative of the waveform) takes on two alternating values which are opposite in sign and different in magnitude, depending on whether the string is sticking to the bow or slipping back from the bow. The abrupt change at the bow from static to sliding friction is practically discontinuous. And such a flexible string in tension supports "all the upper partial tones and their intensity diminishes as their pitch increases" (p. 83). Helmholtz found that the amplitude of the *n*th partial was inversely proportional to the square of *n*.

Helmholtz compared the simplest motion of a bowed string (Figure 2). At any instant the string consists of two straight segments bent at a corner. The projection of the corner moves back and forth with constant velocity on the straight line connecting *a* to *b*. The corner describes in succession two parabolic arcs, which appear to the eye as a lens-shaped envelope. The corner runs around the arcs once in each vibration cycle (440 times/sec for A = 440 Hz), clockwise if the bowing is upward, counterclockwise if the bowing is downward. During most of the cycle, the string sticks to the bow until the corner breaks it away. The string flies back swiftly but is caught again by the bow when the corner returns from the bridge. "Helmholtz' shuttling discontinuity is the timekeeper that precisely triggers the capture and release of the string at the bow" (Schelleng, 1974, p. 70).

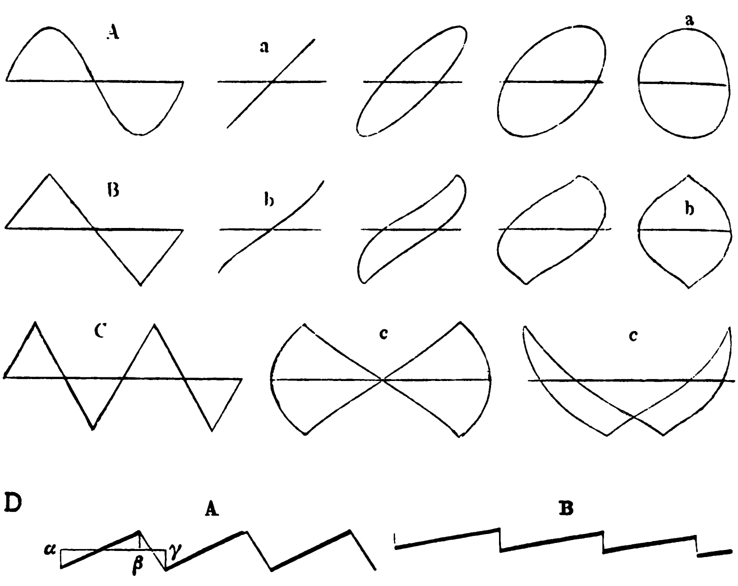

Fig. 1. Vibration microscope images (*a-a*), *b-b*, *c-c*) seen by Helmholtz for sinusoid (A) and for sawtooth waves (B, C). Vibrational waveforms as D (*A* and *B*) were inferred from such observed Lissajou figures as *b-b* or *c-c*. (From Helmholtz, 1865.)

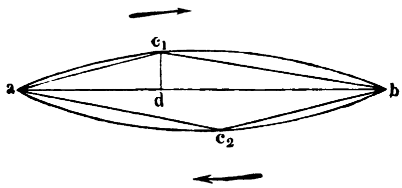

Fig. 2. Shape of the bowed string computed by Helmholtz. (From Helmholtz, 1865.)

Helmholtz believed that the velocity of flyback was constant but as described next, Raman (1918, 1920) showed that this was only approximately true.

Raman's Dynamic Theory of the Motion of Bowed Strings

RAMAN'S NEW METHODS FOR SEEING STRINGS

Raman (1914) wished to go beyond the kinematic knowledge about the motion of bowed strings given by Helmholtz and his successors. His attack was both theoretical and experimental. He sought for a deeper dynamic theory to replace the existing kinematic, geometric models. To this end, he developed a new kinematic method of recording the vibrations of the entire length of a bowed string in one photograph. He also devised a way of projecting and photographing the images which Helmholtz (1865) had seen ". . . by the very beautiful, if a trifle complicated, method of the vibration-microscope" (Raman, 1914, p. 50).

The first question he settled was whether the string's forward velocity is equal to that of the bowed point, a vital issue in any dynamic theory. Raman's (1914) elegant picture (Figure 3) simultaneously recorded both the

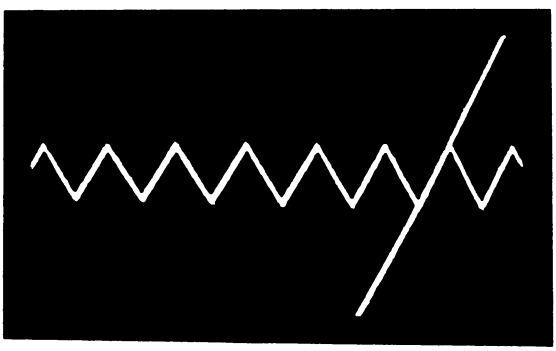

Fig. 3. Simultaneous recording of the bow and the bowed point in contact with it. (From Raman, 1914.)

motion of the bow and the bowed point in contact with it. The motions are absolutely parallel, which led Raman to the generalization "... that in every case in which the motion of the bowed-point is a two-step zig-zag, the velocity of the forward notion is *accurately* equal to that of the bow" (p. 45).

RAMAN WAVES

Helmholtz's shuttling discontinuity is a peculiar form of standing wave. Raman (1918) described the motion in another way, in terms of the component progressive waves of transverse velocity which, happily, take the form of straight lines in which the zigs are slow but the zags are instantaneous. When oppositely moving Raman waves reflected from bridge and nut are summed, the resulting wave shows that two different velocities exist at any point on the string (Figure 4). These two velocities are a function of the position of the discontinuity between sticking and slipping.

The Raman wave drives the bridge with a vibrational force wave whose form is identical to that of the Raman wave. Hence an analysis of the velocity wave reveals the musical quality of the string before it is influenced by

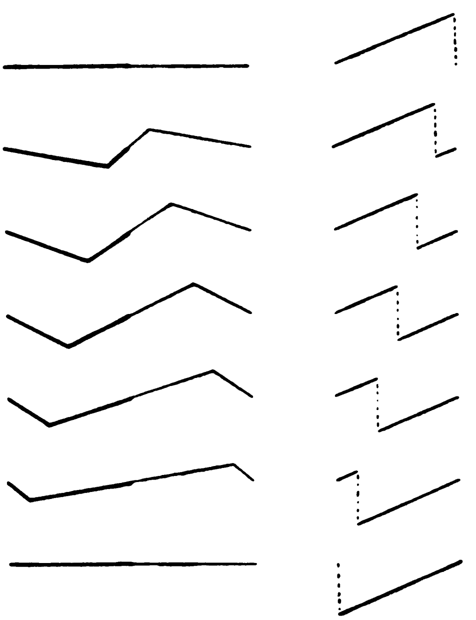

Fig. 4. Successive velocity diagrams at intervals of one twelfth of an oscillation (left column). Vibrations curves and displacement diagrams at equal intervals (right column). The two straight lines passing through the ends of the string meet at the point up to which the discontinuity in the velocity diagram has travelled at any instant (where the light and heavy lines meet). (From Raman, 1918.)

body resonances or radiation. A Raman wave yields a power spectrum in which the amplitude of the *n*th harmonic is $1/n$ times the amplitude of the lowest frequency, for the case in which there is but one discontinuous change of velocity.

Raman (1918) treated a myriad of cases of irrational and rational division of the string by the bowed point, for one, two, or more discontinuous changes of velocity. He displays a great number of curves of velocity, vibration, and frictional force with analyses of their spectra. Raman extends his treatment to many aspects of the bow and bowing, such as frictional properties, finite bow width, interaction with the violin body, and string properties. From this work, he induces a new, kinematic theory of a very important class of transitional vibration that can be controlled by the bow.

ON THE "WOLF NOTE" IN BOWED STRINGED INSTRUMENTS

If one bows a note whose fundamental frequency is coincident with a strong body resonance, of a cello say, a rough pulsating sound, the wolf note, jumps out with its intimations of octave pitch (e.g., Figure 5). Raman's (1916) qualitative explanation assumes that large amounts of energy may be shunted from the Helmholtz motion and stored in vibrations of the body. This further implies that the response of the body lags the bowing in time. While energy is being stored by the body, there is an increase in the rate at which the energy is lost by the string. At the same time the required minimum bow force increases. If this force exceeds the actual bow force before a stable state is reached, the (single-slip) Helmholtz motion transits to a double-slip state. Fed by the stored energy, the second slip grows into a new Helmholtz motion that is out of phase with the old. The repeating cycle is heard as a wolf tone.

Raman's account was essentially correct. It was confirmed and amplified by Schelleng's (1963) elegant, complementary work which used coupled circuits in the frequency domain. One point brought out by Schelleng, but unclear in Raman, was the importance of the reversal of phase in alternate cycles (see McIntyre & Woodhouse, 1978). Both theories agree that the wolf can be tamed by reducing coupling of string and body, which will reduce the maximal amount of energy that can be stored. Thus, a lighter string will reduce coupling or an additional tuned string will lure the wolf away.

Schelleng (1963) explains the wolf tone in terms of the self-excitation of beats, which Cremer (1981/1984, p. 279) flatly rejects. For Cremer, the wolf arises in the nonlinearity of sticking and sliding friction in bowing, which was Raman's (1916) view. This view is supported by McIntyre and Woodhouse (1979), whose simulation of the bowed string properly includes the effect of frictional excitation in the origin of the wolf tone.

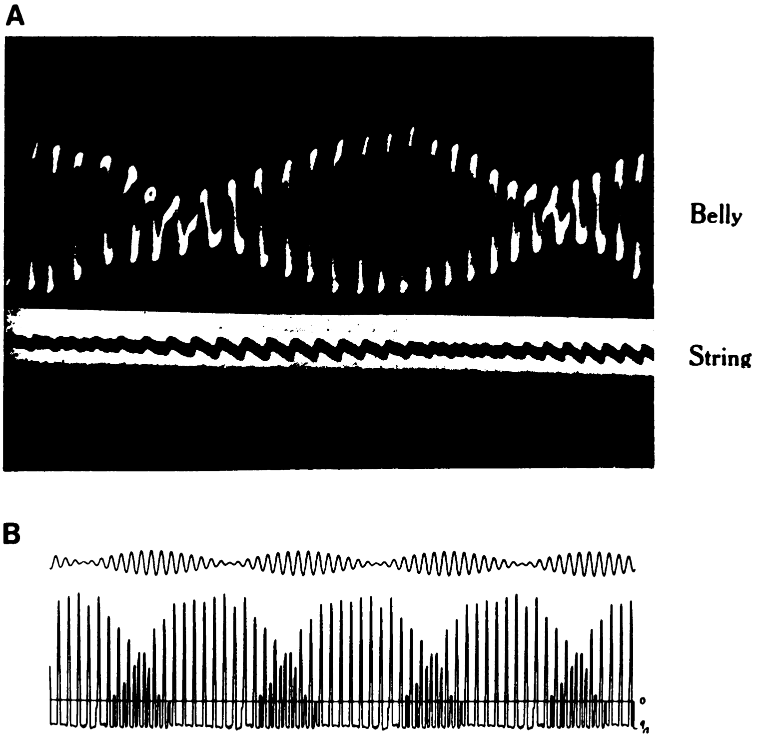

Fig. 5. Simultaneous vibration curves of belly and G string of violincello at pitch of the wolf note (top, from Raman, 1918). Simulation of wolf tones on a computer (bottom, from Cremer, 1981/1984, after McIntyre & Woodhouse, 1979). Velocity at point of bowing is shown in the upper trace; velocity of bridge is shown in the lower trace.

THE BOWED STRING AFTER RAMAN

Raman halted his work on bowed string instruments after 1918 and turned to his professorship at Calcutta University and work on optics.

As part of her history of research on the violin, Hutchins (1983) surveys some 50 years of work on the bowed string, ". . . an extremely nonlinear dynamical system . . ." (p. 1431). She reviews the advances made by using electronic devices like the oscilloscope and recorder. Deeper knowledge of violin systems between 1930 and 1980 led to the development of an integrated octet of new instruments of the violin family of which Hutchins (1967) was the key founder. She was aided in this by Schelleng (1963), who used electrical circuit methods in his violin studies. He followed and illuminated earlier work of Raman and others on the bow-string interaction. In particular, he derived equations for the upper and lower limits of bow force. It turns out that rounding of the Helmholtz corner is a crucial factor in these limits on musical performance (Cremer & Lazarus, 1968).

Cremer's (1981/1984) authoritative book gives a thorough mathematical account of the physics of the violin: the bowing of the string, the body of the instrument, and the radiated sound. (Helmholtzian motion and Raman's model are treated in chapters 3 and 4.) "Only one goal remains elusive: that of deriving credible, objectively measurable criteria for the [subjective] evaluation of instruments" says Cremer (1981/1984, p. 2).

The models of Helmholtz, Raman, and others assumed that the Helmholtz corner was perfectly sharp, but this is not true of a real string because of lossy compliance at the cusps of the Helmholtz motion. The most recent attacks on corner-rounding combine mathematical modeling, experiment, and computer simulation (McIntyre & Woodhouse, 1978; McIntyre, Schumacher & Woodhouse, 1981, 1982). Regular spikes on the bridge-force waveforms, which cause audible noise in musical notes, were shown to arise from slipping of some hairs on the finite-width bow. Raman (1918, pp. 114–118) wrote equations that included a bow of finite width, but said that ". . . a complete evaluation of the integrals and a rigorous detailed treatment do not appear practicable" (p. 118).

Raman's Remarks on Indian Stringed Instruments

RAMAN NOTES A NONLINEAR INFLUENCE OF THE TAMBŪRĀ BRIDGE

Raman (1922) drew attention to the remarkable acoustic properties of the tambūrā and the vīnā. In the tambūrā (Figure 6), "the string passes over the wooden upper surface of the bridge which is curved to shape, and by insertion of a thread of wool or silk, a finely adjustable grazing contact of string and bridge is secured . . . The tones of these instruments show a remarkable, powerful series of overtones which gives them a bright and pleasing quality" (p. 33). The Young-Helmholtz law stated that partials having a node at a plucked point should not be excited. Raman showed by experiments that this law was invalid because forbidden partials were present. To explain "the powerful retinue of overtones," Raman suggested that at or near the grazing point of contact impulses occurred once in each vibration

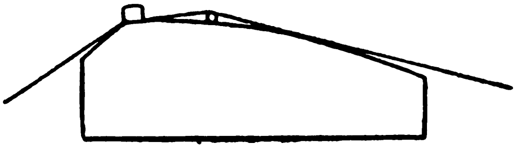

Fig. 6. Curved wooden bridge of tambura. Thread allows adjustable contact of string and bridge. (From Raman, 1922.)

of the string and that there would be a continual transformation of the energy of vibration of the fundamental into the overtones.

RECENT FINDINGS ON SITĀR AND TAMBŪRĀ STRINGS

Recent work confirms that the shimmering, metallic sound of tambūrā, vīnā, and sitār is due to the form of the bridge, which does not have a sharp edge, but is smooth and forms a curved obstacle around which the string wraps and unwraps during its vibration (Burridge, Kappraff & Morshedi, 1982). Variations in string length produce AM and FM modulation sidebands from upper partials. These resultants interact with other strings, giving rise to narrow-band harmonically related clumps (Benade & Messenger, 1982). Such tones have complex time behavior but well-marked pitch. Differences in time of onset or rise time of partials affect timbre of tones of tambūrā (Houtsma, 1982), and violin (McIntyre et al., 1981) and simple abstracted string instruments (Miller & Carterette, 1975). In spite of the nonlinearity of tambūrā strings, partials up to order 30 are very nearly harmonic (Houtsma, 1982; Carterette, Jairazbhoy, & Vaughn, 1988).

ANALYTICAL SOLUTION AND COMPUTER MODELING OF SITĀR STRING

Burridge et al. (1982) give an analytical solution of the sitār (and tambūrā) string as a vibrating string with a one-sided inelastic constraint. The

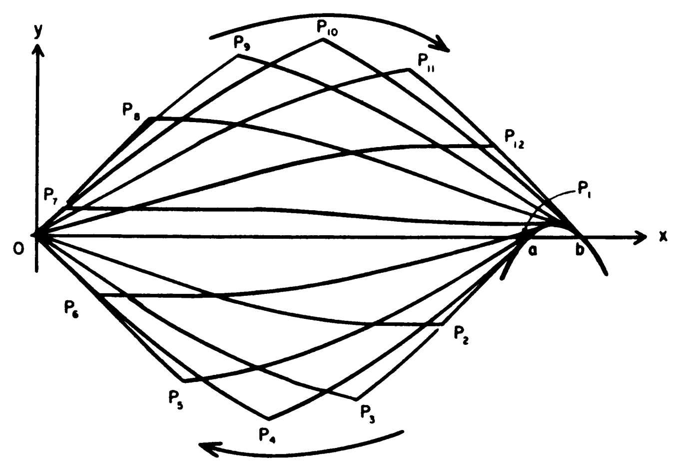

Fig. 7. Twelve positions of the string during a typical first phase cycle. Sequence starts with string horizontal and moving down clockwise from bridge. (From Burridge et al., 1982.)

string behavior was also modeled by computer. They showed that there are two phases to the motion. In the first phase, the plucked string collides with the bridge and is brought to rest instantaneously and inelastically. That part in contact with the bridge does not rebound until the dynamic motion of the string causes an unwrapping from the bridge (see Figure 7). During each vibrational cycle, the straight segment of the string through the origin decreases in slope and impacts the top of the bridge. Helmholtz's first complicated phase gives way to a second phase as equilibrium is approached (Figure 8).

The second phase of motion resembles the solution given by Helmholtz for the motion of a violin string (see Figure 2). The point P moves around the closed curve made by two parabolic arcs with its horizontal velocity equal to the wave speed. At any instant, the position of the string is two straight line segments. The motion of the sitār string is analogous (see Figure 7) except that only one segment of the string is straight; the other is parabolic, albeit rather flat. Furthermore, in Helmholtz's solution cycles are identical and are repeated indefinitely without dissipation. In the case of sitār, the $(n+1)$th cycle will be different from the nth cycle.

We remark that, in the violin, rounding of the circulating corner by periodical reflection at string termini leads to a fall in amplitude of higher partials, which should be perceived as a softer tone quality. However, the corner is resharpened as it passes under the bow (Cremer, 1981/1984, p. 79 ff). And, the greater the bow force, the sharper is the corner. The result is to increase the amplitude of higher partials in the radiated spectrum, which is perceived as an increased brilliance of tone color. Thus, different mechanisms in violin and sitār lead to similar perceptual effects.

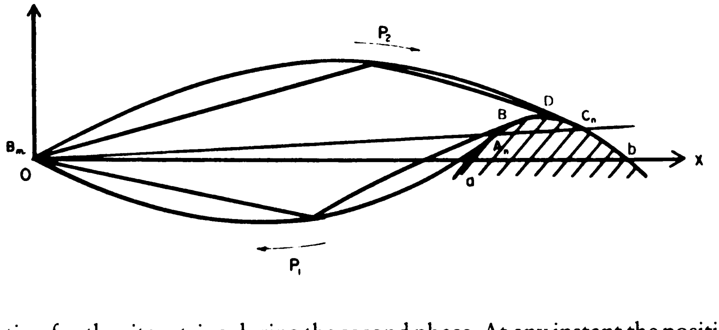

Fig. 8. Solution for the sitar string during the second phase. At any instant the position of the string is given by two segments, one straight (as OP_1, OP_2), but the other parabolic (as P_1B, P_2D). (From Burridge et al., 1982.)

On the Relationship of Tambūrā Spectra to Rāg Scales

SOME NOTES ON INDIAN CLASSICAL MUSIC

Modern North Indian classical music has its roots in ancient Indian music, but appears to have acquired its present form after the fourteenth or fifteenth century. We mention some central features.

1. The main melody line is monodic with an accompaniment.
2. The melody line is generally played against a fixed, unchanged drone which is based on the tonic, its octave, fifth, fourth, seventh, and sometimes the third. This is usually played on a tambūrā, a long-necked lute with four or five strings. The tambūrā has no frets and sounds only on the open-string notes.
3. A vocalist is accompanied by a secondary melody line usually on a sāraṅgī or a harmonium. If on a harmonium, the vocalist himself may play it.
4. There is a percussive line usually played on the tablā. The tablā is a pair of small kettle drums struck with the hands. The percussive instrument serves primarily as a time keeper but also is used for rhythmic variations and improvisation.

We note that many of these musical instruments can do more than one thing. The sitār not only carries a melodic line but has special strings called *eikari* for supplying its own drone and also sympathetic strings call *tarab*, which provide an echo.

In the classical music of India there are two elements: rāg, which is the melodic framework and tāl, which is the time measure.

Rāg

The rāg has no counterpart in Western musical theory. The concept of rāg is that certain characteristic patterns of notes evoke a *rasa* or heightened state of emotion. In fact, the Sanskrit root of rāg is ranj or raj, which means to color to tinge.

The note patterns fuse scalar and melodic elements. Each rāg can be described in terms of its ascending and descending lines and also characteristic figures where certain intervals are emphasized and attention is focused on particular notes.

There are more than 200 extant rāgs. Most are very old and have evolved into their present form. The rāg is a melodic basis for composing and improvisation.

The rāg begins with an ālāp, which is an improvised prelude in free time to exhibit the melodic characteristics of the rāg being performed. When the ālāp is done, a composed piece set in a particular tāl is introduced.

Tāl

The time measure is conceived as a cycle of two main aspects:

1. Quantitative. This is the duration of a cycle in terms of time units of beats, usually three tempos, slow, medium, and fast.
2. Qualitative. This refers to the distribution of stresses or accents within the cycle. There are at least two levels of stress and also an accent by negation where an expected secondary stress in the cycle is omitted. The patterns of stress are often marked during performance by a participant whose role is to keep the tāl.

Notes

There are seven notes: Sa, Re, Ga, Ma, Pa, Dha, and Ni. The addition of accidentals about Re, Ga, Ma, Dha, and Ni gives a set of 12 notes. There is no absolute or fixed pitch attached to the notes. The ground note is called Sa, whatever its pitch. Once the pitch of Sa has been established it remains fixed over the course of the rāg; the other notes of the scale are tuned relative to Sa. There is no formal modulation in Indian music.

The Drone as a Ground in a Figure-Ground Relationship

The rāg and tāl interact against a constant background of the drone. What possible role can the drone play? Jairazbhoy (1971, p. 65) holds that "The ground-note is the point of reference for measuring the intervals used in any rāg." The relation between any note and the ground-note underlies the dynamic quality of the note, a perceptual aspect of which is a tension toward completion. "Only the ground-note is at rest and needs no completion."

However, there are a variety of possible tunings of the tambūrā, and different tunings are chosen for different rāgs. Also, the drone is a dynamic complex of four tones that interact with each other and with the melodic line. The drone is plucked in continual iteration, each string coming to life then decaying in a shimmering tangential dance against the bridge.

TUNINGS

In North Indian music, the tambūrā player alters the degree of grazing contact of the string with the bridge by placing a thread under the string. These threads, which are called juari or "life-giving threads," cause an upward transfer of energy into the higher partials. The resulting spectral complexity is perceived as a buzz.

There are a variety of possible tunings of the tambūrā, and different tunings are chosen for different rags. For example, there are three commonly used tunings, Ni, Pa, and Ma (Table 2). Pa is the most common drone, Ni is the least common and is often used in rags that have neither Pa nor Ma as a primary scale tone. Note that the Ma and Pa tunings are inversions of each other. The man thrust here is to suggest how tunings interact with different rags, based on the Sa tonic.

In order to identify how specific notes of rāg scales are selectively and dynamically enhanced by different drone tunings of the tambūrā, with and without juari, the waveforms of the most common drone tunings of the four strings (Pa Sa Sa Sa′) (where Sa′ = low Sa) were analyzed. From that analysis, we create a model of the projected interaction of those drone tunings and give several musical examples.

SPECTRA OF DRONE STRINGS

Recordings were made of drones and individual strings in the context of a given tuning, played by one of us (NAJ) on a concert-quality tambūrā in a small concert room. The output of a condenser microphone was fed into a single channel of a reel-to-reel tape recorder (Nagra, Model MS) at 19 cm/sec.

Spectra were made on a dynamic signal analyzer (Hewlett-Packard Type 3561A) by using a rectangular window, and samples of either 160 msec or 80 msec (0–2500 Hz or 0–5000 Hz band stops). A stop-band of 2500 Hz was chosen initially because the partials of the string without juari were 40 dB or more below the partial of maximum amplitude by the sixteenth partial. The bandwidth of the analyzer was 23.87 and 47.76 Hz for the 2500-Hz and 5000-Hz band, respectively.

Figures 9, 10, and 11 show the analyses for the three strings Pa, Ni, and Ma, respectively. A comparison of the spectrum of the Pa string without and with the juari thread will illustrate the general findings. Relative amplitude in decibels is plotted on the ordinate with a maximum range of 80 dB, as a function of frequency (in hertz) on the abscissa. The dotted vertical

TABLE 2
The Commonly Used Tunings of the Tambūrā

Tuning	Notes	Degrees	Interval
Pa	Pa Sa Sa Sa′	5 8 8 1	Fifth
Ma	Ma Sa Sa Sa′	4 8 8 1	Fourth
Ni	Sa Ni Sa Sa′	8 7 8 1	Seventh

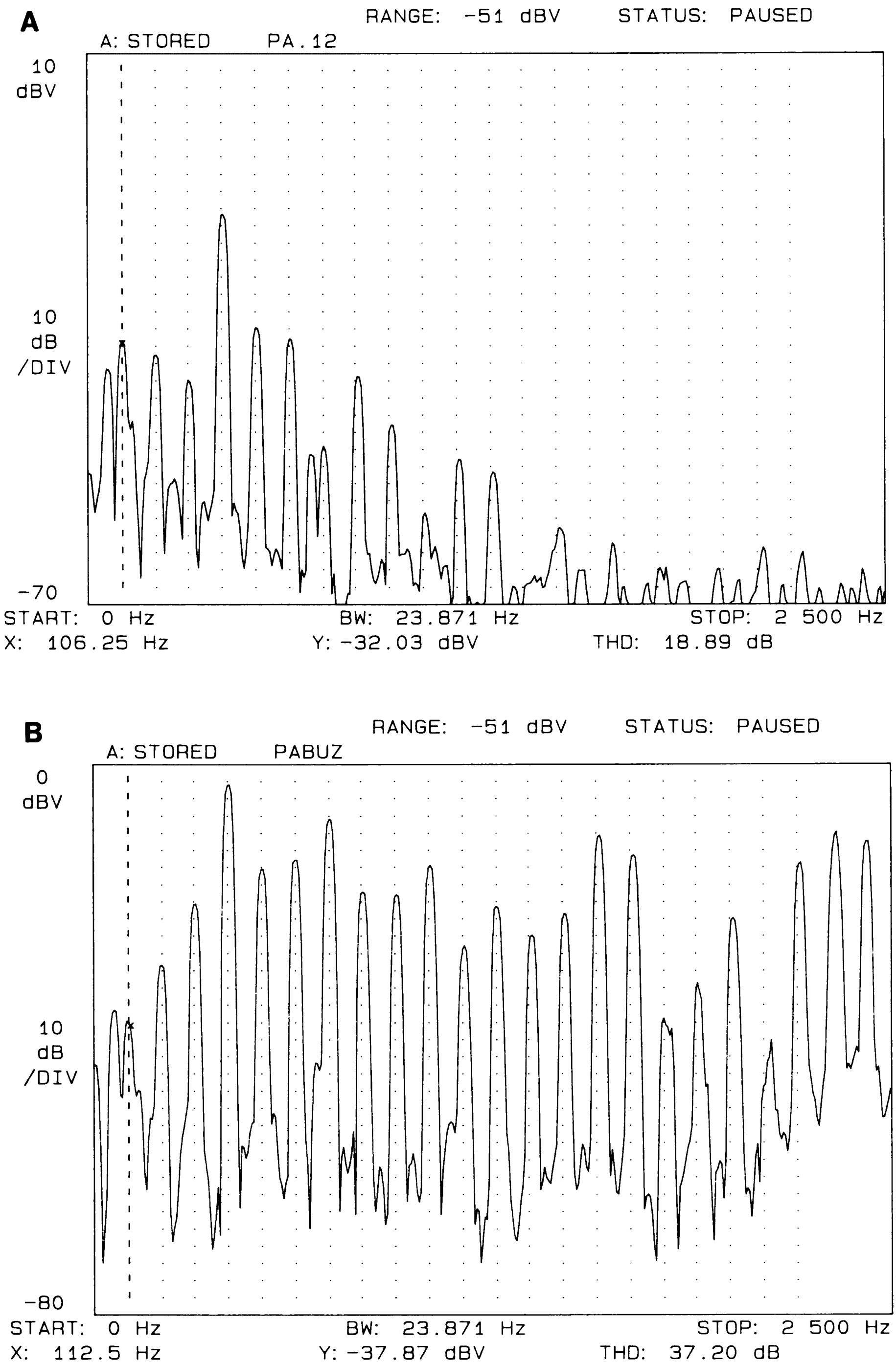

Fig. 9. Power spectrum of Pa drone string without juari thread (A) and with juari thread (B).

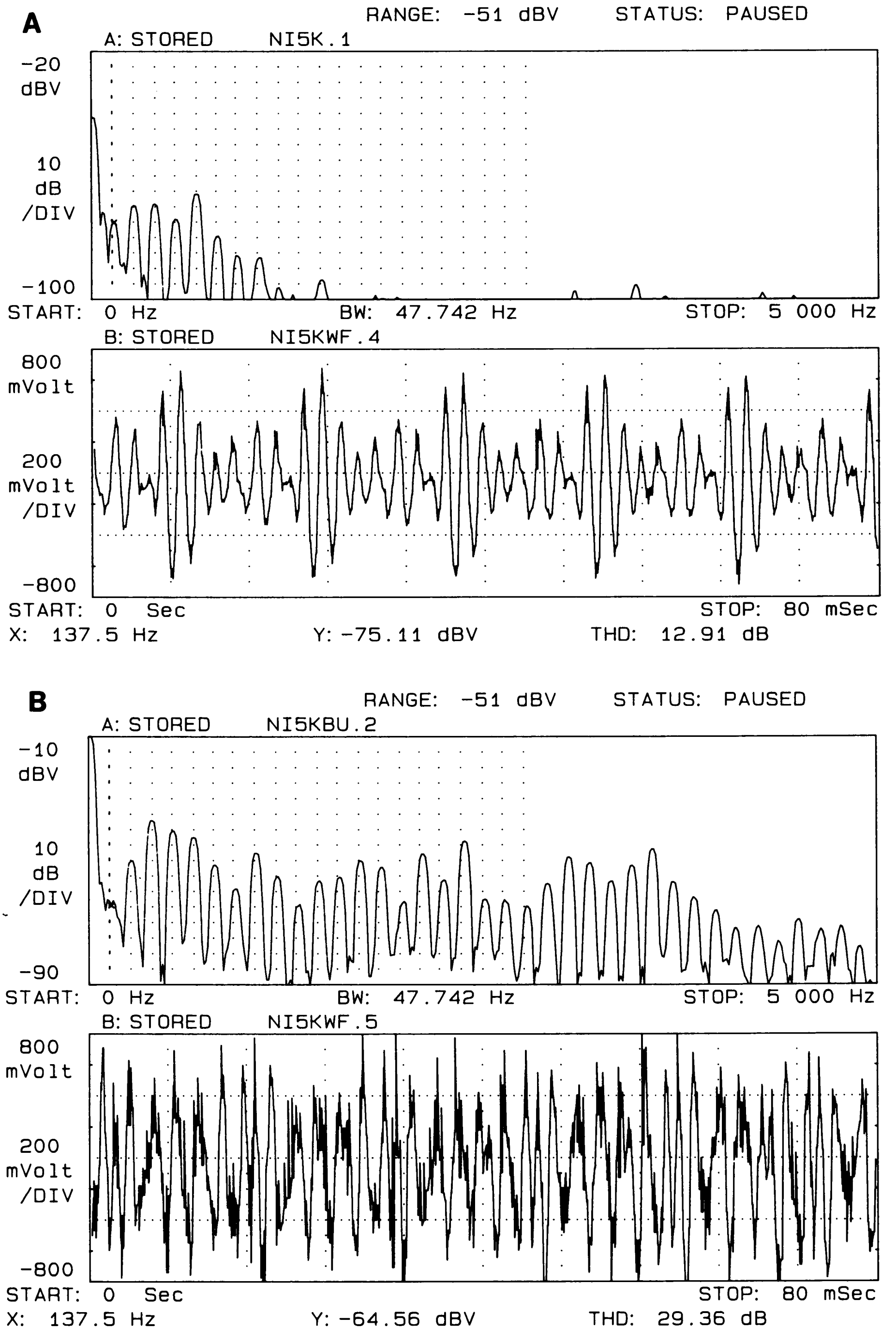

Fig. 10. Power spectrum of Ni drone string without juari thread (A) and with juari thread (B).

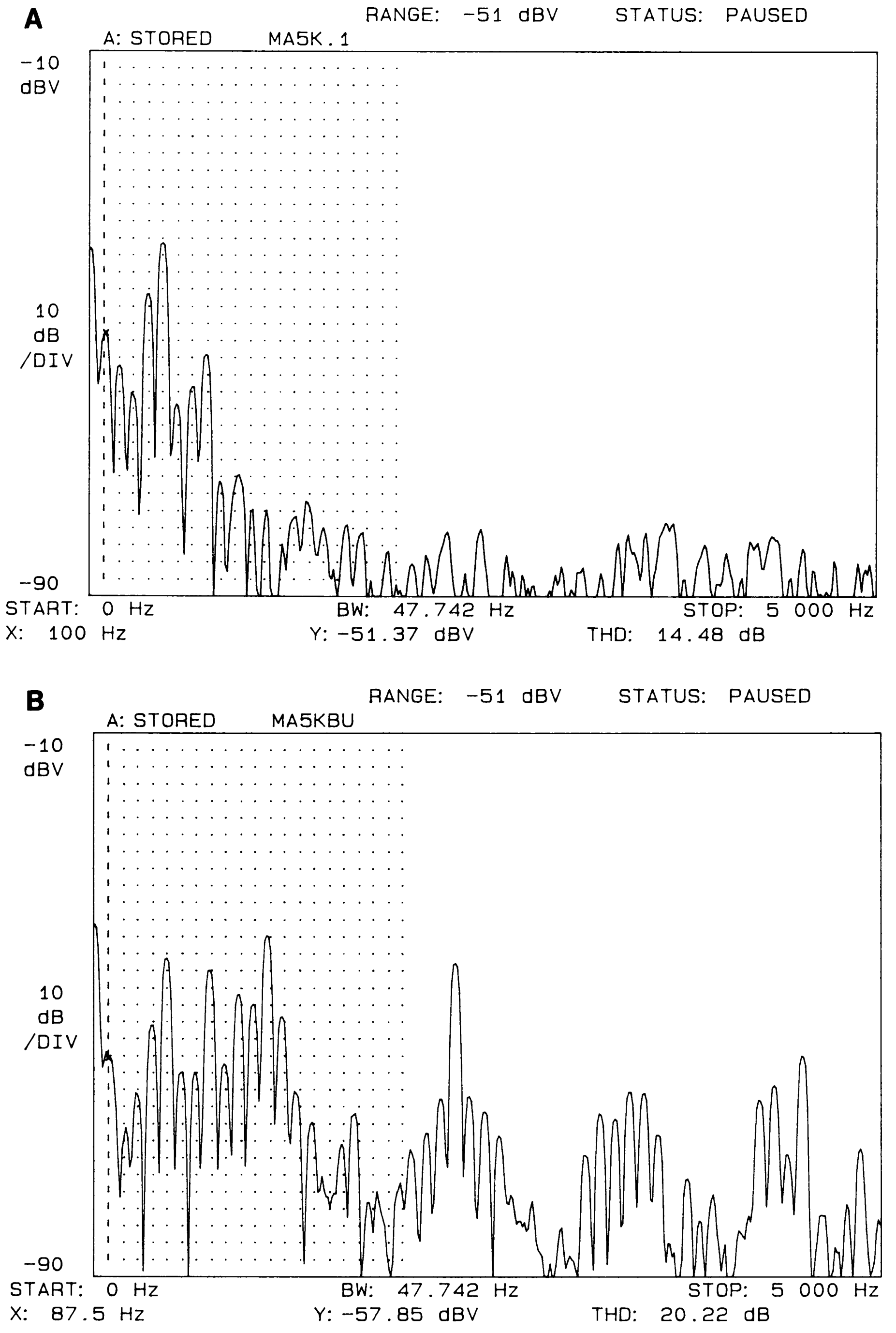

Fig. 11. Power spectrum of Ma drone string without juari thread (A) and with juari thread (B).

lines show the position of the first 20 partials of the fundamental, which is marked by a dashed line. The fundamental is located at 106.25 Hz and has a relative amplitude of −33.17 dBV for the Pa string without juari (Figure 9A). Total harmonic distortion (THD) was 18.10 dB, which corresponds to about 804% rms voltage distortion. [THD = 20 log (% rms/100)]. The Pa spectrum is typical of the other spectra without juari, namely that the second and third partials tend to be weaker than the fundamental and the fourth and fifth tend to be strongest, with a nearly linear decline in decibels thereafter. Resolving power is lost at about the fourteenth or fifteenth partial, with the remaining spectrum being essentially noise.

Do not be misled by the term "harmonic distortion," which is just a measure of the relative amounts of energy in those partials that are integral multiples of the fundamental frequency. High harmonic distortion here implies a massive transfer of the energy of vibration of the fundamental into the harmonic partials, which agrees with Raman's (1922) view.

The spectral analysis of the Pa string with juari thread (Figure 9B) is in marked contrast to that without juari. The fundamental is still at 106.25 Hz, of course, but its relative amplitude is quite low. The amplitudes of the second and third partials increase steeply to the fourth partial, which tends to be the component with maximal amplitude. The next strongest partials are the seventh, tenth, fifteenth, and sixteenth, and partials 21 and 22 have considerable amplitude. This extended range of high-amplitude partials is reflected in the total harmonic distortion of 37.27 dB, which corresponds to a total harmonic rms voltage distortion of over 7300%.

Spectral analyses of the Ni string (131.25 Hz), as well as 160-msec segments of the waveforms, without and with juari, are shown in Figures 10A and 10B, respectively. The spectral analyses for the Ma string (93.75) without and with juari are shown in Figure 11A and 11B.

RELATING TAMBŪRĀ SPECTRA TO MUSICAL SCALES

The 21 ratios (scale degrees or notes) of just temperament (Taylor, 1965) may be assigned to the set of 12 tones from which rāg scales may be formed, as shown in Table 3. We relate these 21 values to the Indian scale on the basis of tunings used by musicians of the culture. The measurements of Deva (1967), Jairazbhoy and Stone (1976), as well as our own measurements of the tambūrā strings have been considered in determining the distribution of the 12 scale tones over the 21 degrees. These measurements verify that intervallic distances between tones of the scale may be identified in the justly tempered division of the octave. Where there is a variation of tuning between performers or performances the difference consistently falls within a window that is acknowledged in the just-temperament system to span the range of a given tone. For example, the ambiguous areas that sur-

TABLE 3
Assignment of Scale Tones According to the Ratios of Just Intonation

Degree	Name	Ratio	Frequency	Cents
1	Sa	1	70.00	0[a]
2	Re-komal	25/24	72.91	71
3	Re-komal	16/15	74.67	112
4	Re	9/8	78.75	204
5	Ga-komal	75/64	82.00	275
6	Ga-komal	6/5	84.00	316
7	Ga_1	5/4	87.50	386
8	Ma_1	125/96	91.00	457
9	Ga_2	32/25	89.60	427
10	Ma_2	4/3	93.00	498[a]
11	Ma-tivr	45/32	98.40	590
12	Ma-tivr	36/25	100.80	631
13	Pa	3/2	105.00	702[a]
14	Dha-komal	25/16	109.00	773
15	Dha-komal	8/5	112.00	814
16	Dha	5/3	116.67	884
	Dha	12/7	119.44	925
17	Ni-komal	225/128	123.00	997
18	Ni-komal	9/5	126.00	1018
19	Ni_1	15/8	131.25	1088[a]
20	Ni_3	125/64	136.70	1159
21	Ni_2	48/25	134.40	1129
22	Sa′	2	140.00	1200[a]

[a]Measured tones of the three tambura tunings.

round the minor and major third (300–400 cents) and the flat and natural seventh (1100–1100 cents) degrees of a musical scale are taken into account. One discrepancy found was the possibility of a sixth degree (Dha) which could be played as 119 Hz instead of 116. This tone fills a gap between the sixteenth and seventeenth tones of the justly tempered division of the octave. For the purposes of our analysis, the Dha tone was allowed the two values it assumes in practice.

To the end of relating tambūrā spectra to musical scales, we devised an index of interaction complexity based on the number of possible coincidences of a spectral peak with each of the 21 notes of the justly tempered system. A coincidence match was declared only if two conditions held: (1) the amplitude of the peak was no less than 10 dB below that of the fundamental and (2) the frequency of the peak was within ±25 cents (a window of 50 cents) of the frequency of a scale note or its higher harmonics over all octaves of the musical range (< 5 kHz). Each match was weighted by its relative amplitude in decibels and summed for a given degree. Call this sum the weighted number of matches. The weighted index of complexity, S, was defined as the sum of the weighted number of matches over all 21 degrees. An algorithm for computing S is given in the Appendix.

Values of weighted index of complexity were computed for tambūrā strings without juari and with juari. The results for the Ma drone string are shown in Table 4 (no juari) and Table 5 (juari). The usual assignment of note names to each degree is shown. An example will show how the weighted index is enhanced by the addition of the juari thread. Sa (Degree 1, Table 4) without juari has nine matches whose weighted sum is 381, but the number of matches for Sa with juari (Degree 1, Table 5) increases to 53 and has a weighted sum of 2129. The weighted richness over all 21 degrees without juari is about 13,507 but with juari increases to about 37,980. Of the total possible 1597 matches, there are 905 with juari but only 287 without juari. This increase of complexity when the juari thread is added is shown in Figures 12 and 13, which plot the relations among amplitude (in decibels relative to the fundamental), partial number, and scale degree (musical note) for the Pa drone.

In looking at these data it is vital to know that a drone is the stable ground against which a melody is interwoven through the harmonic struc-

TABLE 4

Ma Drone String (93 Hz) without Juari Against the Scale of Sa (280 Hz)

Degree	Name	Matches	Weighting	Avg. Weight	dB Diff.	Avg. dB Diff.
1	Sa	9	381.39	42.38	−52.32	−5.81
2	Re-komal	16	757.80	47.36	−13.24	−0.83
3	Re-komal	16	736.78	46.05	−34.26	−2.14
4	Re	15	700.31	46.69	−22.54	−1.50
5	Ga-komal	16	727.78	45.49	−43.26	−2.70
6	Ga-komal	16	729.02	45.56	−42.02	−2.63
7	Ga	6	271.92	45.32	−17.22	−2.87
8	Ma	28	1343.68	47.99	−5.64	−0.20
9	Ga	5	232.43	46.49	−8.52	−1.70
10	Ma	28	1343.68	47.99	−5.64	−0.20
11	Ma-tivr	7	333.67	47.67	−3.66	−0.52
12	Ma-tivr	9	457.17	50.80	23.46	2.61
13	Pa	13	629.09	48.39	2.62	0.20
14	Dha-komal	15	685.81	45.72	−37.04	−2.47
15	Dha-komal	11	494.11	44.92	−35.98	−3.27
16	Dha	11	580.67	52.79	50.58	4.60
17	Ni-komal	15	715.83	47.72	−7.02	−0.47
18	Ni-komal	18	838.02	46.56	−29.40	−1.63
19	Ni	10	476.88	47.69	−5.02	−0.50
20	Ni	12	553.38	46.11	−24.90	−2.07
21	Ni	11	517.61	47.06	−12.48	−1.13

NOTE. The weighted richness of this relationship is 13507.03. The average weighted richness is 643.19. Total possible matches = 1597. Actual matches = 287. This relationship has an absolute measure of richness equal to 0.18.

ture of the sound. So, when we say that a certain rāg scale and drone tuning interact strongly, we mean that the partial structure within the drone influences the melodic structure of the rāg. Thus, we infer from an emphasis of specific partials by a drone string that a performer will hear those tones more easily than those that are less well represented. Both performer and listener will expect these points of strong interaction as points of resolution.

The musical significance of juari is easier to grasp if the 21 scale degrees are assigned to the seven notes of North Indian classical music. These are Sa, Re, Ga, Ma, Pa, Dha, and Ni, of which Sa and Pa are fixed. The other five are movable. These are Re-komal, Ga-komal, Ma-tivr, Dha-komal, and Ni-komal, where ***komal*** and ***tīvr*** are analogous to flat and sharp, respectively, of Western music. We make this assignment for all drone strings in the way shown for Ma in Table 4.

In the case of the Pa drone string (106.25 Hz) with and without juari, Figure 14A shows the strength of interaction (in mean decibel difference) summed across all partials for each of the 12 musical notes (Sa . . . Ni). Clearly the interactive strengths have increased, often markedly so, as with Ma, Ma-tīvr, and Dha-komal. In the case of the other drone strings, there is

TABLE 5

Ma Drone String with Juari (93 Hz) Against the Scale of Sa (280 Hz)

Degree	Name	Matches	Weighting	Avg. Weight	dB Diff.	Avg. dB Diff.
1	Sa	53	2129.15	40.17	−106.92	−2.02
2	Re-komal	37	1647.94	44.54	86.91	2.35
3	Re-komal	45	1948.15	43.29	49.60	1.10
4	Re	45	1871.28	41.58	−27.27	−0.61
5	Ga-komal	45	1845.28	40.01	−53.27	−1.18
6	Ga-komal	51	2092.00	41.02	−59.69	−1.17
7	Ga	36	1427.53	39.65	−91.31	−2.54
8	Ma	57	2471.34	43.36	66.51	1.17
9	Ga	22	828.36	37.65	−99.82	−4.54
10	Ma	54	2347.35	43.47	69.09	1.28
11	Ma-tivr	37	1513.31	40.90	−47.72	−1.29
12	Ma-tivr	38	1607.03	42.29	3.81	0.10
13	Pa	52	2087.99	40.15	−105.89	−2.04
14	Dha-komal	44	1817.09	41.30	−39.27	−0.89
15	Dha-komal	38	1575.90	41.47	−27.32	−0.72
16	Dha	39	1740.43	44.63	95.02	2.44
17	Ni-komal	42	1775.50	42.27	3.52	0.08
18	Ni-komal	47	2039.19	43.39	−56.26	1.20
19	Ni	38	1655.74	43.57	52.52	1.38
20	Ni	54	2239.31	41.47	−38.95	−0.72
21	Ni	31	1319.67	42.57	11.78	0.38

NOTE. The weighted richness of this relationship is 37979.54. The average weighted richness is 1808.55. Total possible matches = 1597. Actual matches = 905. This relationship has an absolute measure of richness equal to 0.57.

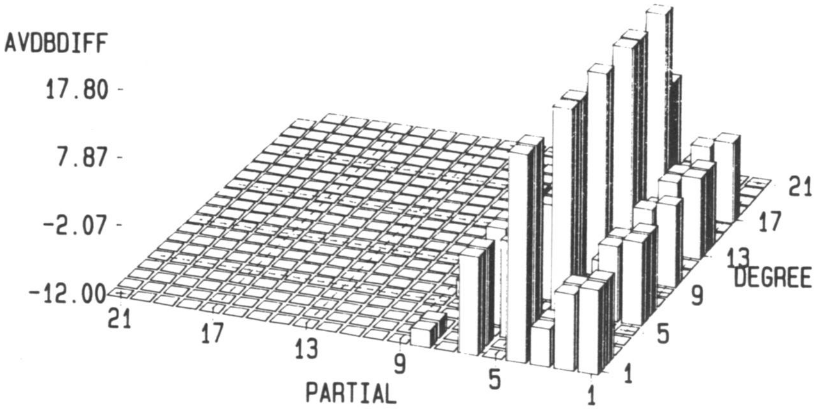

Fig. 12. Plot of amplitude, partial number, and scale degree of Pa drone string (106.25 Hz) without juari.

likewise an increase in interactive strengths when the juari is added, and the patterns of the increased strengths are different for each drone (Figures 14B,C). These unique patterns are what drive the choice of different drone tunings for different rāgs.

FROM CIRCLE OF THĀT TO RĀG

So far, we have related the spectra of the drone strings, in different tunings, to the possible notes of a scale, based on Sa, which has been divided into 21 or 12 values, according to a musical scale theory. We see that there is a significant difference in the strengths of certain partials when the juari threads are placed on the bridge. The threads modulate the behavior of the string, which is already nonlinear because of its grazing contact with the curved bridge (Raman, 1922).

We wish to predict some possible consequences of the juari for the Indian musical system and to see how the effect of adding the juari might influence choices of the sitārist.

How is the actual music related to the components of certain rāg scales? Good clues may be found in the larger structural system in Indian music known as the Circle of Thāt(s). This is a system of classifying groups of rāgs according to their scales and the placement of accidentals (altered notes, as in the Western chromatic scale). There are 10 commonly used thāts (of 32 possible), each having seven pitches in the sequence Sa Re Ga Ma Pa Dha and Ni, or altered versions of those tones with komal (flat) and tīvr (sharp). In considering the system of thāts, no account is taken of melodic motives as in rāg, nor is any emotional quality associated with a thāt (Jairazbhoy, 1971, p. 46).

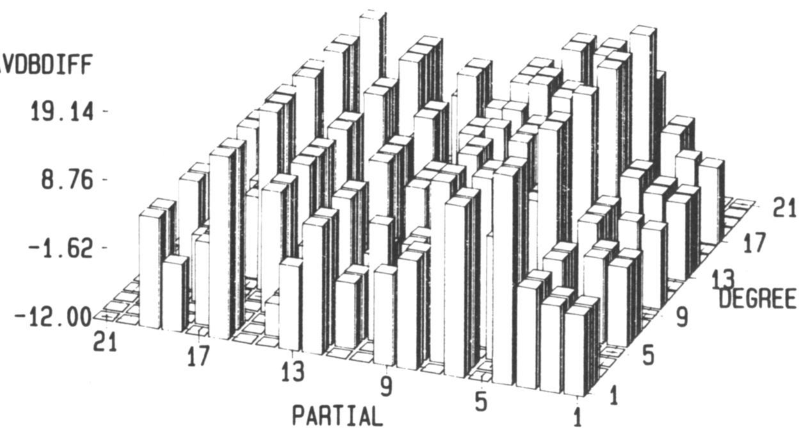

Fig. 13. Plot of amplitude, partial number, and scale degree of Pa drone string (106.25 Hz) with juari.

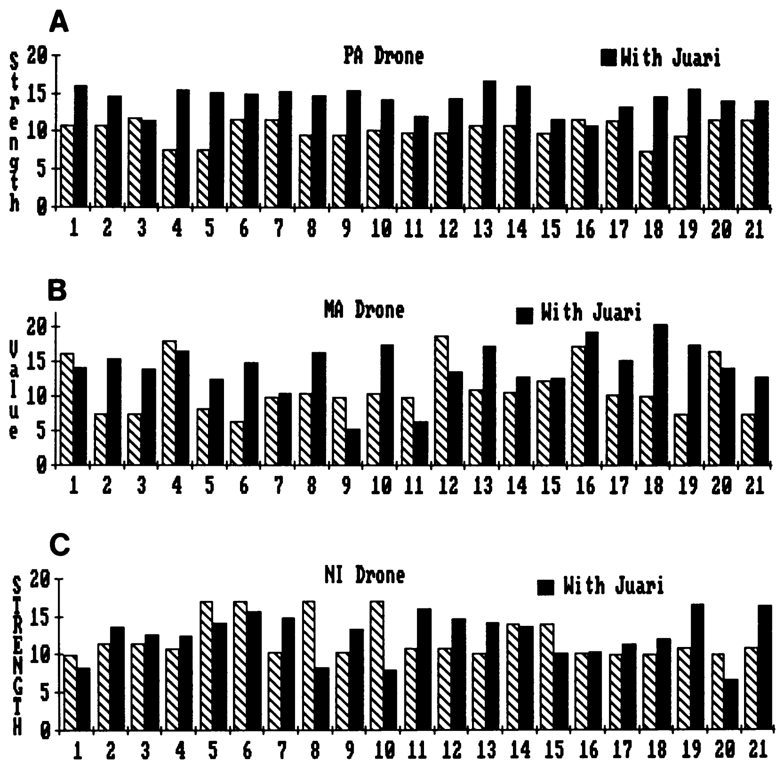

Fig. 14. Strength of interaction in mean decibel difference relative to the fundamental, summed across all partials for each of the 21 degrees (see Table 3). Black bars are without juari; hatched bars are with juari. (A) Pa drone string, (B) Ma drone string, (C) Ni drone string.

The oldest of the 10 thāts are identical to the seven modes used in Gregorian chant, sometimes referred to as the Church modes. Three alterations of those seven modes have been added in India since the seventeenth century, and their use has coincided with the appearance of tambūrā drone tunings other than the more traditional Pa-Sa. The Church modes roughly correspond to the white notes on a modern piano, but were most likely sung in an unequal tuning system by using the intervals of the Pythagorean or justly tempered scale. Each mode starts on one of the seven diatones beginning on middle C of the piano. Thus C1-C2 is the Ionian mode, now known as the major scale; A1-A2 is the Aeolian mode, which is the natural minor scale; and so on. In the Indian musical system, each thāt is transposed so that the starting note is always the tone Sa. The intervals for each thāt are preserved by the addition of the accidentals *tīvr* (sharp = ♯) and *komal* (flat = ♭).

A few examples will clarify this notion. The scale of Kāfi Thāt (Sa Re Ga-komal Ma Pa Dha Ni-komal) corresponds to the Dorian mode (D1-D2). Bhairvī Thāt is exactly the Phrygian mode (E1-E2) and Mārvā Thāt is very close to what is known as the "Gypsy" scale wherein the harmonic minor scale is altered. Mārvā is actually an alteration of the Lydian mode (F1-F2) except for a flatted second degree. Figure 15 shows the scales of the three thāts with intervals in cents. Observe that all semitones are wider than equal temperament, but with a compensating narrowness in some whole tones and thirds, which maintains the 1200-cent octave.

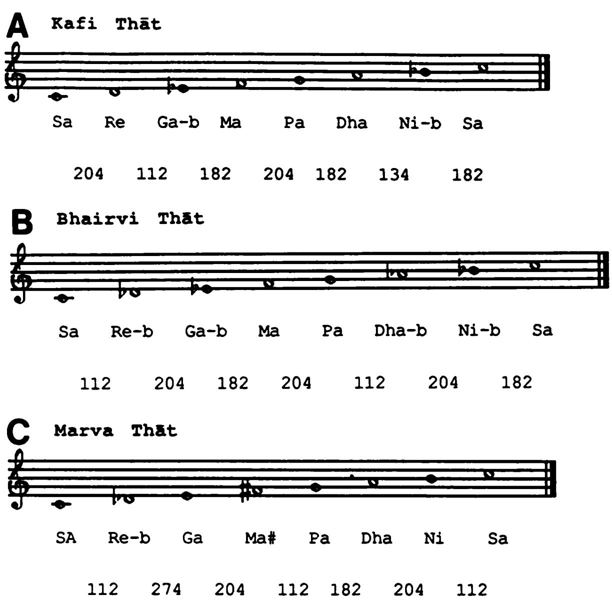

Fig. 15. Scales of three thāts in Western notation. The Indian name is shown below each note, along with interval size in cents.

There are certain natural relationships to consider when comparing the components of a rāg scale to specific drone tunings and to its thāt. For example, the Ma-Sa drone tuning would not be used ordinarily with a rāg belonging to any of the group of thāts that are identified by the use of a Ma-tīvr. However, we have seen that because of the acoustic effects elicited by juari, the tambūrā drone has the potential for complex and subtle interaction with the sitār.

AN EXAMPLE IN KĀFĪ THĀT

Is there a change in the correlation of a given drone tuning with a particular rāg or thāt when the juari threads are present? We compare two rāgs associated with Kāfī Thāt, Bhīmplāsī and Bāgeśrī. Rā Bhīmplasī has the form given by the notes {Sa Re Ga-komal Ma Pa (Dha) Ni-komal}. (The tone Dha is often omitted as a member of the scale.) Rag Bāgeśrī is based on the scale tones {Sa Re Ga-komal Ma (Pa) Dha Ni-komal} and generally omits Pa.

On the basis of measured spectra for single strings without juari, model predictions show that the Ma drone fits the notes of Kāfī Thāt well, but the Pa drone does not. (The Ni drone would not be used against a that having Ni-komal.) When the juari are added, the weighted interaction complexity of both Ja and Pa increases greatly.

This means that Ma-Sa and Pa-Sa have stronger affinities to the scale of Kāfī Thāt with juari than without (see Figure 16). In the cases of the two rags, we predicted from the model that the Ma drone fits the scale of Bāgeśrī well (Figure 17), that the Pa drone fits the scale of Bhīmplāsī well (Figure 18) and that these affinities are heightened by the effect of juari. Indeed, this prediction is borne out in practice. (In Figures 17 and 18 only the obligatory notes of the rāg are shown.)

The case for use of Ma-Sa with Bāgeśrī and use of Pa-Sa with Bhīmplāsī can be computed from relations between the partials and the scale tone being enhanced by the juari threads. Juari effects work in conjunction with other factors, including the constraints of the musical system and aesthetic preference of the improvising performer.

FULL-DRONE TUNINGS IN SEVERAL THĀTS

In musical practice, the tambūrā drone is made of all four strings and three notes sounding together, with juari threads adjusted for maximal buzz. To the end of relating full-drone tunings to given thāts, we analyzed the three drone tunings with all strings sounding:

{Pa Sa Sa Sa′} {Ma Sa Sa Sa′} {Sa Ni Sa Sa′}

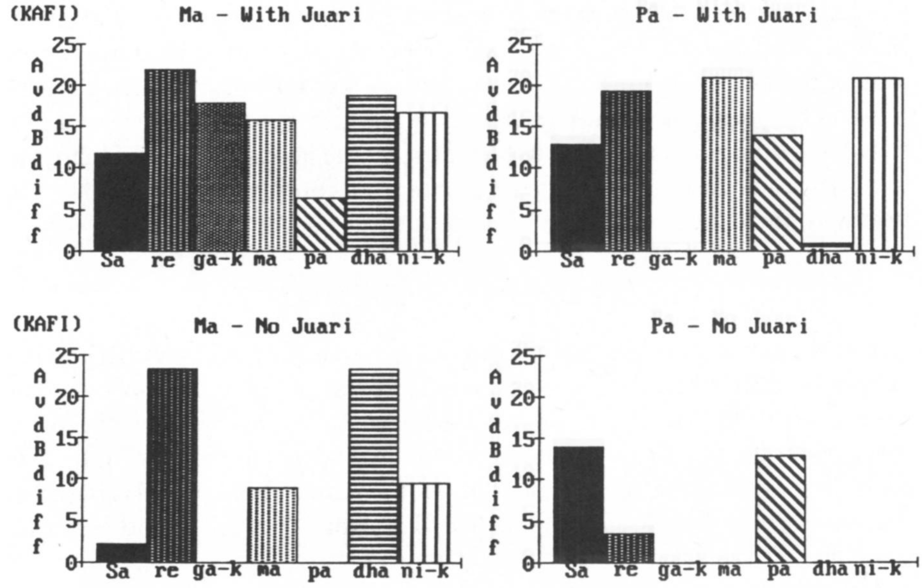

Fig. 16. Strength of interaction in mean decibels relative to the fundamental summed across all partials for the notes of Kafi Thāt for Ma and Pa drone strings with and without juari.

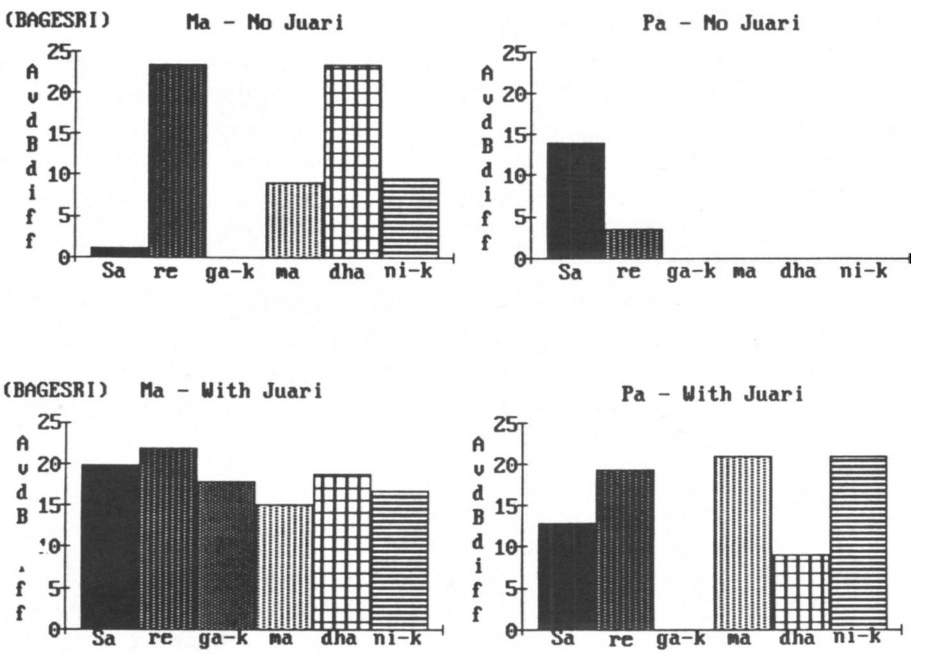

Fig. 17. Strength of interaction in mean decibels relative to the fundamental summed across all partials for obligatory notes of Rāg Bāgeśrī for Ma and Pa drone strings with and without juari.

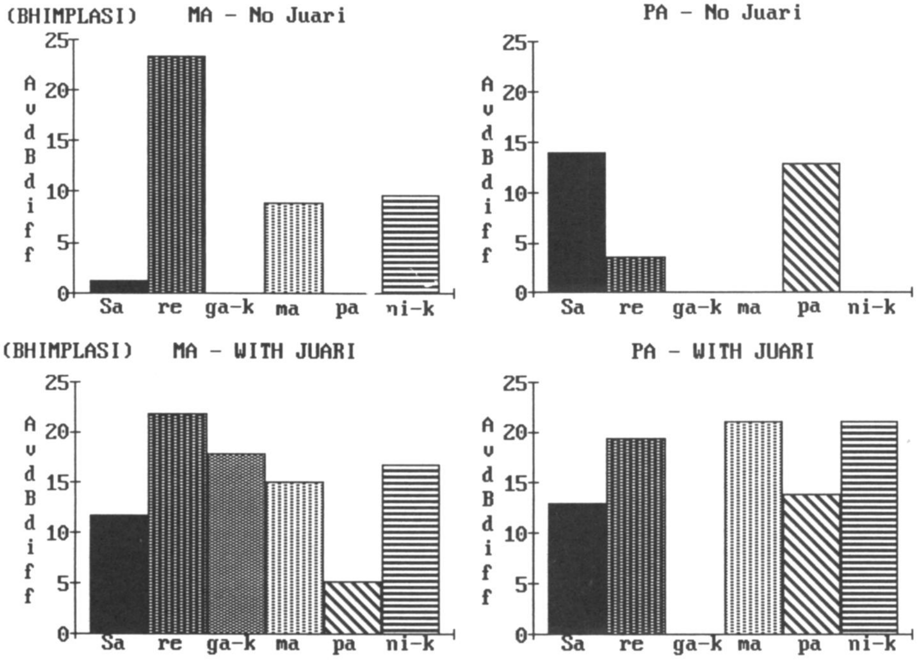

Fig. 18. Strength of interaction in mean decibels relative to the fundamental summed across all partials for obligatory notes of Rāg Bhīmplāsī for Ma and Pa drone strings with and without juari.

Figures 19, 20, and 21 show the power spectra of the tunings for Pa, Ma, and Ni, respectively. As in the case of single strings, the use of juari causes greatly increased power in the upper partials. Tables 6–8 show the values of the weighted index of complexity for the three tunings as previously discussed for the case of single string (see Tables 4 and 5). As before, we assigned the 21 scale degrees to the 12 notes (seven plus five accidentals) of the Indian scale.

Figure 22 displays the interaction of each of the three tunings with Kāfī, Bhairvī, and Mārvā thāts, derived from Table 6.

Kāfī Thāt (Figure 15A):

Sa Re Ga-komal Ma Pa Dha Ni-komal

Bhairvī Thāt (Figure 15B):

Sa Re-komal Ga-komal Ma Pa Dha-komal Ni-komal

Mārvā Thāt (Figure 15C):

Sa Re-komal Ga Ma-tīvr Pa Dha Ni

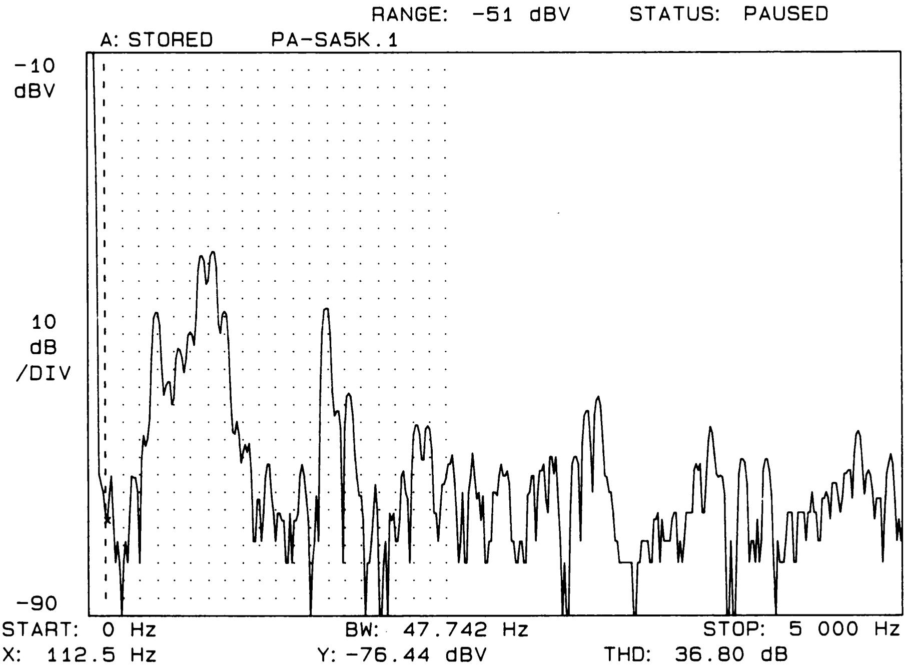

Fig. 19. Spectrum of full-drone tuning {Pa Sa Sa Sa′}.

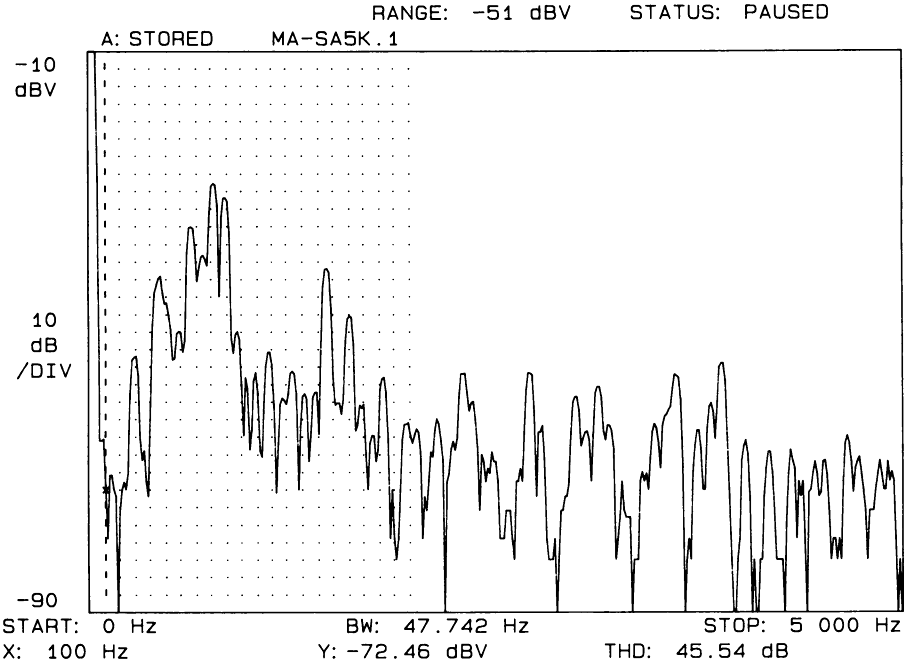

Fig. 20. Spectrum of full-drone tuning {Ma Sa Sa Sa′}.

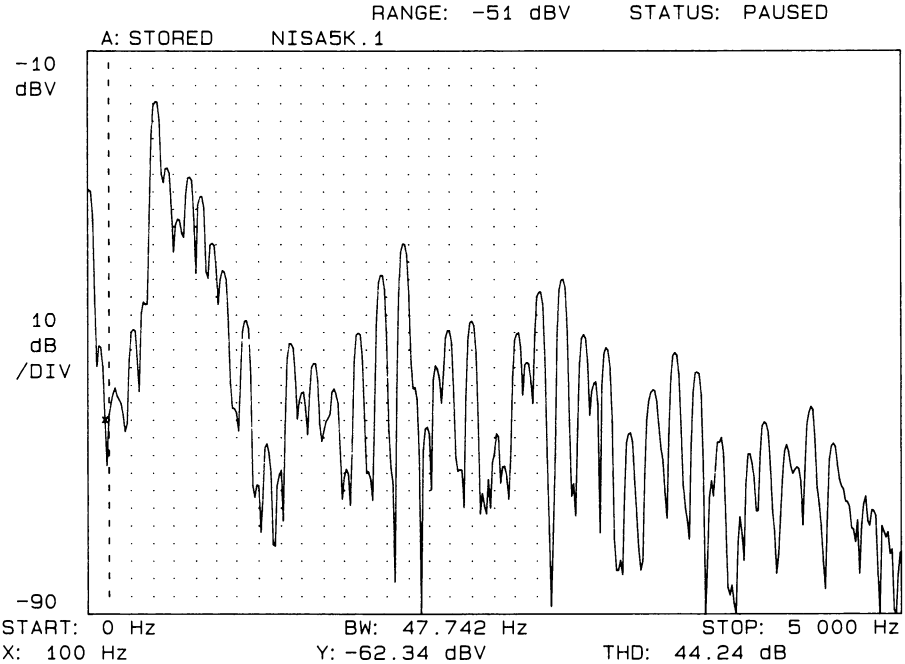

Fig. 21. Spectrum of full-drone tuning {Sa Ni Sa Sa′}.

The central comparison to be made here is of the tunings for a given thāt. Down the column of Kāfī Thāt, notice that the Pa, Ma, and Ni tunings have distinctly different emphases on particular notes of the thāt. Of course, the expected strengths of Pa and Ma are evident for the Pa and Ma drone tunings. Re and Ga-komal are much stronger in Pa tunings than in the Ma tuning. The tones Dha and Pa are relatively very strong in the Ni tuning, albeit the presence of Ni-komal rules out the use of the Ni tuning with this thāt.

In Bhairvī Thāt (middle column), the Pa tuning has a relatively weak interaction with Re-komal and Ga-komal. In the rāg Bhairvī, Re-komal is sometimes replaced with Re natural; the Dha-komal may be altered also. This suggests that the Pa tuning is better for rāg Bhairvī, because the relation with Re-komal is secondary here. A case could be made for the Ma tuning with a rāg in this that which emphasizes the relation between Re-komal, Ma, and Ni-komal. The pentatonic rāg Malkos, in Bhairvī Thāt, is usually accompanied by the Ma drone. Jairazbhoy (1971) notes that there have been performances of Mālkoś wherein the Sa drone note is abandoned entirely so that Ma becomes the ground tone. The graph of the Ma drone tuning suggests that there is an acoustic, perceptual basis for this variant performance (Fig. 22, row 2, column 2).

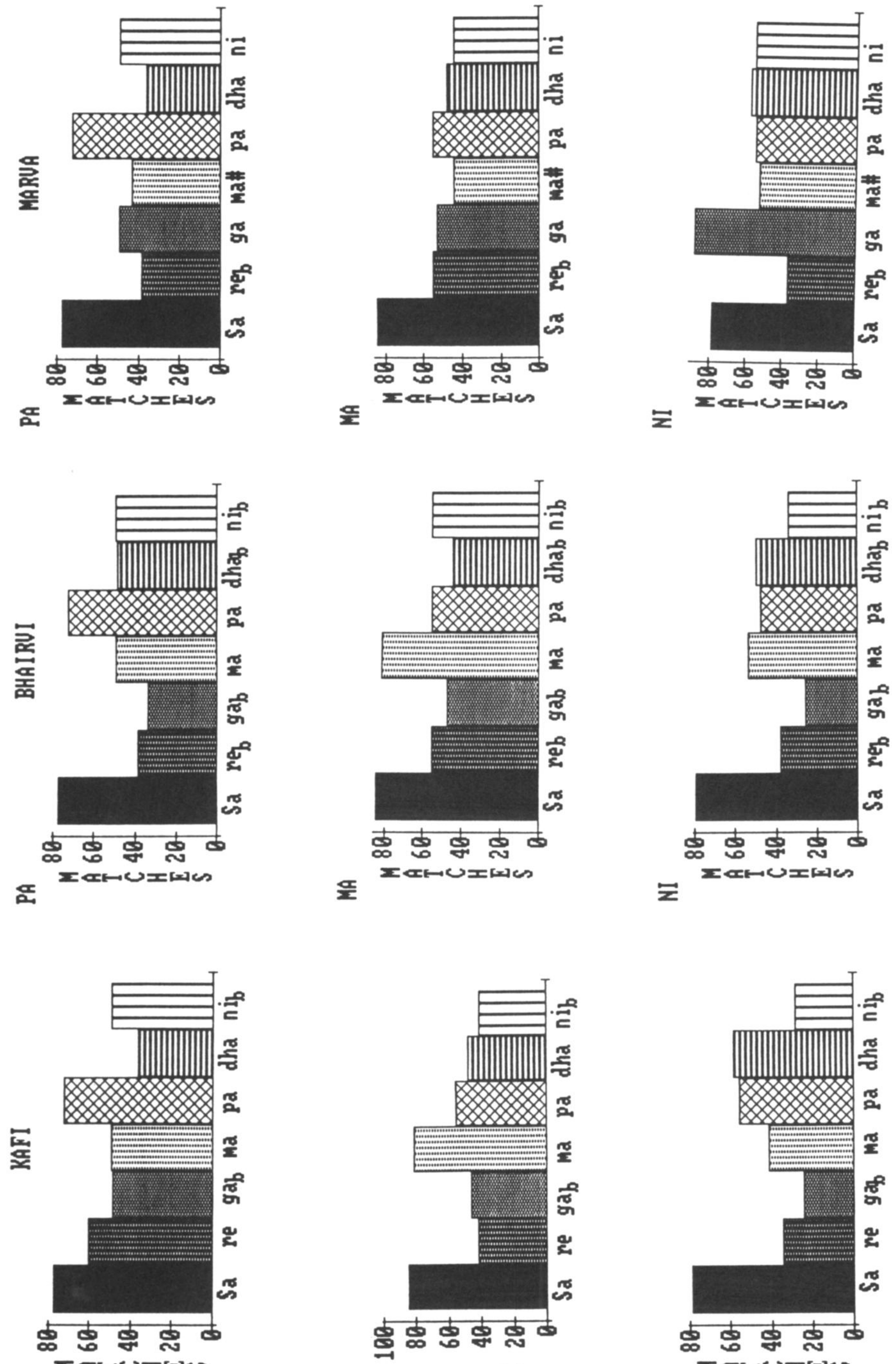

Fig. 22. Interaction of the three thāts (columns) with each of the three full drone-tunings (rows).

In Mārvā Thāt (right column), the rāg Mārvā uses the tones Ma-tīvr and Ni but omits Pa. There is also a strong affinity of Ga and Ni in rāg Mārvā, which uses the Ni tuning, and this is confirmed by the strength of Ga in our analysis (row 3, column 3).

Our goal in this article has been to exhibit an analytical basis for the choice of tunings in the choice of rāgs, rather than to make strong specific claims. The perceptual basis for the efficacious choice of tunings in North Indian classical music must be tested experimentally, for example by preference scaling studies, among cultures and between musicians and nonmusicians.[1]

TABLE 6

Pa (105 Hz) Full Drone Compared with Scale of Sa (280 Hz)

Degree	Name	Matches	Weighting	Avg. Weight	dB Diff.	Avg. dB Diff.
1	Sa	78	2822.82	36.19	705.35	9.04
2	Re-komal	26	694.66	26.72	19.27	0.74
3	Re-komal	39	1273.29	32.65	212.48	5.45
4	Re	61	2007.11	32.90	410.57	6.73
5	Ga-komal	34	1026.63	30.19	109.27	3.21
6	Ga-komal	49	1643.14	33.53	353.53	7.21
7	Ga_1	50	1741.87	34.84	394.52	7.89
8	Ma_1	31	990.97	31.97	167.98	5.42
9	Ga_2	39	1257.80	32.25	246.79	6.33
10	Ma_2	50	1866.92	37.34	519.57	10.39
11	Ma-tivr	44	1549.88	35.22	391.27	8.89
12	Ma-tivr	45	1598.35	35.52	390.30	8.67
13	Pa	73	2649.21	36.29	758.19	10.39
14	Dha-komal	26	866.98	33.35	150.09	5.77
15	Dha-komal	49	1659.57	33.87	332.61	6.79
16	Dha	37	1333.03	36.03	342.05	9.24
17	Ni-komal	48	1738.92	36.23	453.10	9.44
18	Ni-komal	50	1924.07	38.48	593.32	11.87
19	Ni_1	50	1598.99	31.98	280.69	5.61
20	Ni_3	15	466.14	31.08	77.29	5.15
21	Ni_2	37	1204.65	32.56	259.32	7.01

NOTE. Weighted richness = 31915.00, Total average weight = 709.18, Total dB difference = 7167.56, Total average dB difference = 151.26.

1. We thank Dr. Carleen Hutchins, Permanent Secretary of The Catgut Acoustical Society for her gracious help in alerting us to some crucial issues in musical strings. The UCLA Libraries helped us by finding original works of Raman (Engineering & Mathematical Sciences Library) and of Helmholtz (History Division of the Biomedical Library). Professor Roger A. Kendall, Department of Ethnomusicology and Systematic Musicology, made thoughtful comments on the manuscript.

TABLE 7
Ma (93 Hz) Full Drone Compared with Scale of Sa (280 Hz)

Degree	Name	Matches	Weighting	Avg. Weight	dB Diff.	Avg. dB Diff.
1	Sa	85	3783.94	44.52	1575.88	18.54
2	Re-komal	37	1511.96	40.86	851.19	23.01
3	Re-komal	56	2272.41	40.58	1021.66	18.24
4	Re	43	1843.35	42.87	777.13	18.07
5	Ga-komal	48	2068.47	43.09	1027.87	21.41
6	Ga-komal	47	2080.66	44.27	986.47	20.99
7	Ga_1	54	2123.96	39.33	810.09	15.00
8	Ma_1	68	2959.69	43.52	1779.24	26.17
9	Ga_2	23	982.63	42.72	408.74	17.77
10	Ma_2	82	3497.80	42.66	1811.90	22.10
11	Ma-tivr	45	1857.96	41.29	876.89	19.49
12	Ma-tivr	49	2035.86	41.55	872.61	17.81
13	Pa	56	2586.02	46.18	1269.18	22.66
14	Dha-komal	45	1899.95	42.22	918.88	20.42
15	Dha-komal	58	2533.72	43.68	1125.79	19.41
16	Dha	49	2127.33	43.41	1074.23	21.92
17	Ni-komal	56	2375.64	42.42	1102.86	19.69
18	Ni-komal	42	1815.47	43.23	739.72	17.61
19	Ni_1	45	1871.03	41.58	801.84	17.82
20	Ni_3	44	1913.51	43.49	1253.36	28.49
21	Ni_2	31	1253.77	40.44	668.00	21.55

NOTE. Weighted richness = 45395.13, Total average weight = 893.92, Total dB difference = 21753.53, Total average dB difference = 428.17.

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2. Hutchins's (1983) history of research on the violin was very helpful for preparing this article, as were a number of the original papers on the musical acoustics of the violin family reprinted in Hutchins (1975, 1976). Also very useful was McIntyre and Schumacher (1978).

TABLE 8

Ni (131.25 Hz) Full Drone Compared with Scale of Sa (280 Hz)

Degree	Name	Matches	Weighting	Avg. Weight	dB Diff.	Avg. dB Diff.
1	Sa	79	3938.35	49.85	638.90	8.09
2	Re-komal	17	742.12	43.65	48.77	2.87
3	Re-komal	38	1744.50	45.91	165.60	4.36
4	Re	35	1720.06	49.14	258.81	7.39
5	Ga-komal	30	1390.04	46.33	147.54	4.92
6	Ga-komal	25	1193.38	47.74	143.63	5.75
7	Ga_1	89	4200.68	47.20	537.73	6.04
8	Ma_1	22	1038.12	47.19	136.02	6.18
9	Ga_2	16	736.58	46.04	73.78	4.61
10	Ma_2	42	2067.13	49.22	310.03	7.38
11	Ma-tivr	54	2663.70	49.33	446.00	8.26
12	Ma-tivr	20	981.89	49.09	134.89	6.74
13	Pa	56	2645.45	47.24	326.65	5.83
14	Dha-komal	23	1069.39	46.50	110.74	4.81
15	Dha-komal	34	1604.91	47.20	176.21	5.18
16	Dha	59	2799.38	47.45	366.93	6.22
17	Ni-komal	36	1756.77	48.80	266.97	7.42
18	Ni-komal	29	1426.90	49.20	216.95	7.48
19	Ni_1	57	2542.79	44.61	193.44	3.39
20	Ni_3	23	1073.46	46.67	136.81	5.95
21	Ni_2	5	209.34	41.87	−3.41	−0.68

NOTE. Weighted richness = 37544.94, Total average weight = 990.23, Total dB difference = 4832.99, Total average dB difference = 118.20.

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Appendix

The level of interaction of the spectra of the drone strings with the scale based on SA is computed as follows:

$$\text{Match}\ (d_{ik}, \zeta_p) \equiv \frac{\zeta_p}{2^{\frac{25}{1200}}} \leq d_{ik} \leq \zeta\, 2^{\frac{25}{1200}}$$

and

$$a_p \geq a_0 - 10$$

$$s_{ik} = \begin{cases} a_p \text{ if match } (d_{ik}, \zeta_p) \\ 0 \text{ otherwise} \end{cases}$$

$$s_i = \sum_{k=1}^{20} S_{ik}$$

Where d_i is base frequency of scale degree i, $i \in 1...21$
d_{ik} is kth multiple of d_i, $k \in 1...20$
ζ_p is frequency of pth partial, $p \in 0...29$,
δ is frequency resolution,
ζ_0 is base frequency of measured drone string,
$\zeta_p = \zeta_o + p \cdot \delta$,
a_p is measured amplitude in decibels at pth partial, and
s_{ik} is strength of scale degree i, multiple k.